Classroom are *not* the ER

Taking the time kids need to appreciate mathematics

January 5, 2016

Abstract

We just want to assure the reader that classrooms are not emergency rooms where somebody will bleed to death if teachers don't cover enough material today. We'll show, on a few examples, how to teach some results, concepts, algorithms that will promote kids' long term understanding and appreciation of mathematics while also acquiring the necessary skills to handle technical stuff like algorithms.

Contents

1 The main observations

1.1 Take time

I think the biggest mistake math teachers make at any level, K-12 or in college, is that they *don't take enough time* with a concept, result, or formula. For one reason or another, teachers rush ahead to cover more material than the kids can handle or are willing to put up with. We are not talking about here not practicing a formula or result enough, but making sense of them, understanding them, appreciating them, and, in little ways, even helping kids to discover them.

1.2 The two main goals of math

It's extremely important that kids eventually understand two things about mathematics.

1. Math originates from the world around them, and not some abstract construction smart people invented to torture kids with.

2. The reason for using math is to describe the world around them in a *simple* way, and be able to *predict* how things will work. *Abstraction* is simplification and clarification, *not* complication.

The basic principle, not unique to teaching math, is that the kids need to *enjoy* doing math. If only a few nerdy kids in a class enjoy math, and the rest just tag along or are lost, the teaching must change.

1.3 Proof $\neq$ Understanding

Click here to glimpse into a typical Common Core friendly explanation of geometry from a guy named Wu at Berkeley who is a big promoter[1] of CC math. It's enough to read the preface to understand that mathematicians now think, kids will share their taste: "a need for a proof". Nothing can be further from the truth. Even physicist don't share mathematicians' taste

[1] Click here to read his article, written with world famous mathematician Frenkel, that appeared in the Wall Street Journal. Frenkel really is a great mathematician, but he has no clue what the general audience of math can or should digest. His "best selling" book, *Love & Math* is as unreadable for a layman as Stephen Hawking's books, like *The brief history of time*. That was also a best seller, but I met nobody who read it through or understood it. I certainly understood very little of it, though I did read it through couple of times.

for precise proofs. Why? *Because a mathematical proof doesn't always give a good enough, intuitiv explanation why a result is true.*

A mathematical result just has to make sense to kids, and they need to be able to appreciate it. They don't always want to or need to know why a result is true. We can enjoy the warm sunshine in the spring without wanting to know why the sun is hot, and we certainly can appreciate honey in our hot tea against cold without needing to know that, in fact, it was regurgitated from the bees' stomachs.

Perhaps the most basic reason kids don't like math is because they feel insecure in a math class. Even if they are good at solving calculus problems, they feel, all these results, formulas, concepts were invented by much smarter people than they are, and they have no opportunity to look behind the scenes to see how discoveries are made.

Well, this is exactly what we try to show here: *how to make kids appreciate mathematics and be part of the discovery process.*

1.4 Be interactive

The best way to check on kids' understanding of math is to talk to them. This needs to be done *all the time*: ask questions, invite their questions, answer their questions, and *let kids talk to each other*.

In the examples below, we sometimes suggest explicit questions to ask. But even when we say "do this, do that", the teacher should refrain from telling kids how things are. Kids should do their own discoveries in their own pace.

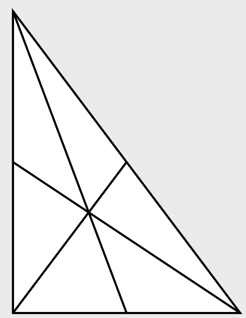
2 Result: A striking example from triangle geometry

2.1 Purpose

The purpose here is to teach kids about the center of gravity of a triangle. The standard way of stating the result is,

Theorem 2.1 (Existence of center of gravity in a triangle)**.** *In a triangle, connect each corner to the midpoint of the side opposite to it.*

Then the resulting three line segments meet at a single point inside the triangle.



The result sometimes is stated as

> The three medians of a triangle are concurrent at a point called the centroid.

While seeing theorem 2.1 as it is written above makes even high school kids flinch, I think the whole section can be taught to 5th or 6th graders. Let's see if you agree.

2.2 Incorrect ways: rushing

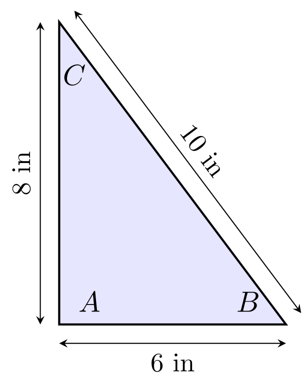
There are two main existing ways of teaching, and both are excused by quoting time constraints. The first way is the CC way: the teacher with very little preparation, just states the result, and then attempts to prove it, assuming, the proof will take care of understanding. The other method is when the teacher states the result, does not prove it with the assumption that the kids won't understand it, anyways, and instead, crams in lots of applications of the result, leaving many as homework problems.

Typically, in both cases, the teacher may even attempt to squeeze in another result or concept into the class. In college, the prof may even try to squeeze in four more similar results.

2.3 Taking time

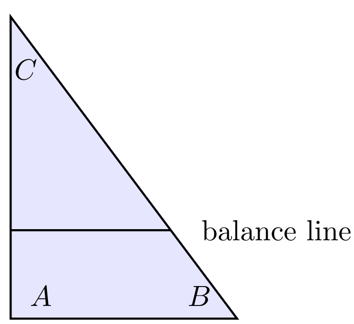
Kids have no idea what's going to be covered today. They will discover and then formulate the result together.

Kids will be paired up (or more kids can work together), and intially each pair will get a ruler, and a large enough triangle cut out from card board. The sidelengths of the triangle should be even numbers, for example, 6in, 10in, 8in will do. The corners of the triangle are labeled by A, B, C, as on this picture



2.3.1 Balance the triangle on the edge of the ruler

Ask the kids to try to balance the triangle on the long edge of the ruler. Once succeeded (it may take as much as 5 minutes), one kid should hold the triangle in balanced position while the other kid in the pair should mark the two sides of the triangle where the ruler meets them. Then connect these marks with a straight line. They may get something like,

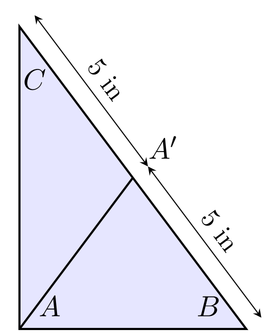


The difficulty with balancing the triangle on the edge of the ruler, with the ruler going through two sides of the triangle is that with a new unmarked triangle, it's difficult to find the balance again.

Can we somehow speed up the balancing act?

2.3.2 Balance with ruler going through a corner

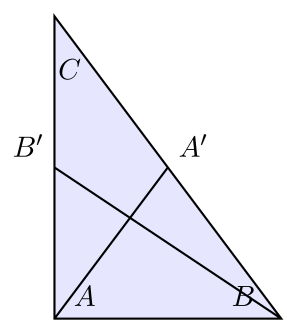
Tell the kids to erase the "balance line" from their triangle. Then ask them to balance the triangle on the edge of the ruler so that the ruler goes through one of the corners. Those who balanced through corner A, then marked and drew the balance line, they'd have



Some kids might remark that it seems that the balance line goes through the midpoint A' of the line segment BC. If none of them remark that, ask the kids to try to figure out where the balance line meets the line segment BC. To make sure that their observation is correct, they can verify it by measuring the BA' distance to be 5in which is half of the 10in BC distance.

2.3.3 Balance while ruler goes through another corner

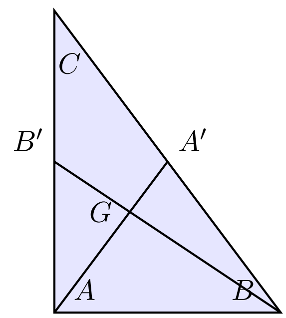
Ask the kids to connect the midpoint of another side, say, AC, with the opposite corner.



Ask them now to balance the triangle by placing the ruler on the BB' line. (So what happened now, they used math to figure out how to balance instead of experimenting)

2.3.4 Balance the triangle with one finger

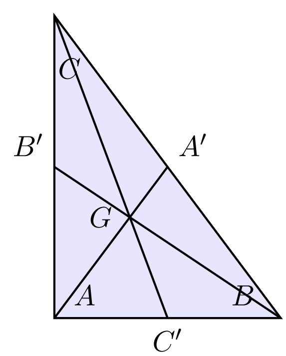
Ask the kids to balance the triangle on the tip of their index finger. They (well some, for sure) will be amazed to see that the point where they can balance the triangle is exactly the point of intersection G of the lines AA' and BB'.



2.3.5 Where will the remaining balance line go through?

The kids can then be asked: "*Where do you think the remaining balance line, starting at C, will go? Will it also go through G?*"

They probably will guess that it will go through G, and they will proceed to verify it by connecting the midpoint C' of AB with C, and seeing that the line does go through G.



Pretty amazing stuff!

2.3.6 Summary

Kids can be asked the question,

"What did we figure out today?"

. I suspect, they'll mostly mention finding the balance point, but with little guidance, they'd also say that the balance lines all go through the same point.

Behind the scenes, we demonstrated for them that we can draw a rather simple mathematical conclusion from our observation of the physical world, and then we can use the mathematics we learnt to find a solution to *all* similar physical problems.

Notable is the fact that so far no discussion of *why* a balance line works, why we can really balance the triangle if we put the ruler along it. I don't really expect kids being interested in addressing this while doing the balancing experiments, so there is no need to bother them with it. It can wait till the next class. Let them enjoy the moment.

I remark that the mathematical proof of theorem 2.1 is rather short, but most kids don't get it. But the proof is not needed for the kids to have a good understanding of the result and have a feeling about what it really says. If this is the case, it will be difficult for them to forget the result.

Despite the lack of precise proof, they were doing mathematics, and exactly the way mathematicians or physicists are doing it: conducting experiments.

2.3.7 Homework

I am not a big fan of homework, but if it's necessary, the kids could be asked to challenge a family member or friend to balance a triangle, they cut out from a cardboard, on the tip of their finger or a match. They could even make it a competition about who can find the balance point faster.

To make it easier on the kids, you could give them triangles to take home.

2.3.8 Next class

To warm the kids up and remind them about what they did last time, hand out papers with three triangles on each, and ask the kids to find the balance point for each triangle, and also draw all three balance lines, not just two to find the balance point.

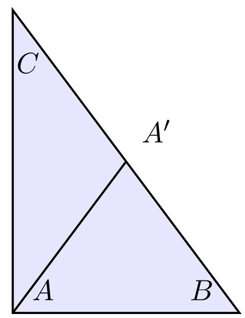
Now they can be asked *why* a balance line works:

"Why do you think the line going through A is exactly the one which connects A with A', the midpoint of the opposite side BC?"

For most kids, this may not be the most interesting question—especially in grade 5. So I don't think there's a need for a long discussion. To help guide them a bit, one could ask

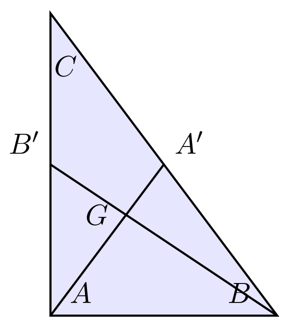
"Do you think it's difficult to balance a triangle out in Space?"

If they have no idea about the connection, they could be reminded of weightlessness. Then they can be guided to trying to compare the areas (weight) of the two triangles ABA' and $AA'C$,



They can then use their knowledge of how to find the area of a triangle to conclude that the two triangles have the same area, hence, from a balancing viewpoint, the same weight.

The topic of balancing can be concluded with some discussion of why the intersection G of two balance lines must be the balance point,



Maybe the term "center of gravity" can be introduced as a reminder that all this business with balancing makes sense only here on Earth but not out in Space.

Then one can move on to some other material, such as finding the center of gravity for other planar objects, like a rectangle, but, unless the kids demand this, it may be better to discuss something different.

2.3.9 Challenging problems

Kids can think about the following problems in class or as homework problems after the second class. I wouldn't assign them all at once. Too much. Personally, I'd ask the questions distributed to several classes when I am already covering different material.

While answering some of the questions could be difficult to some of the kids, they all can experiment with them. And that's what I want a kid in college be willing and able to do: experiment.

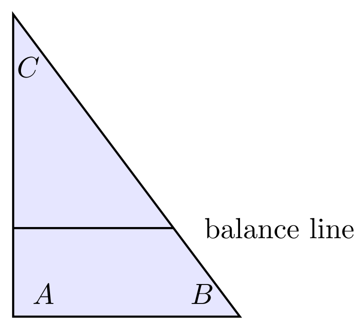
Some may find it difficult to understand the difference between problems 2.2 and 2.6.

Problem 2.2. Are all lines going through G balance lines?

Problem 2.3. Is it possible that two balance lines are parallel to each other?

Problem 2.4. Is is possible that two balance lines don't intersect each other inside the triangle?

Problem 2.5. Does the first balance line you found go through G?



Problem 2.6. Do perhaps all balance lines go through G? Can you find a balance line that doesn't go through G?

Problem 2.7. Can you find another balance point, other than G, in the triangle?

Problem 2.8. Where do you think a rectangle's balance point is?

Problem 2.9. Can you find 4 balance lines for a rectangle? Any more?

Problem 2.10. Where do you think a disk's balance point is?

Problem 2.11. Can you find 4 different balance lines in a disk? How many more can you find?

Problem 2.12. Using your ruler, compare the lengths of AG and GA'. Then compare the lengths of BG and GB'. Finally, compare the lengths of CG and GC'. What do you notice?

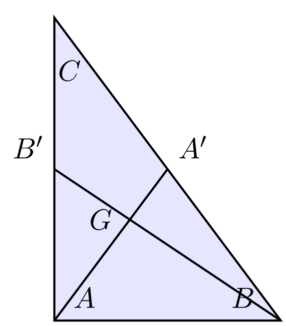
Problem 2.13. How can you find the midpoint of a line segment using ruler and compass only?

2.3.10 Sketch of proofs

There are two significantly different proofs of theorem 2.1: one using coordinate geometry, the other one belongs to elementary geometry. Neither of them are complicated, but neither will reveal why we can balance the triangle at its center of gravity.

Proof using coordinate geometry. The idea of the proof is to simply write down the equations of the balance lines, and verify that they all intersect at the same point. It will turn out that the coordinates of the center of gravity is the arithmetic average of the coordinates of the corners of the triangle. □

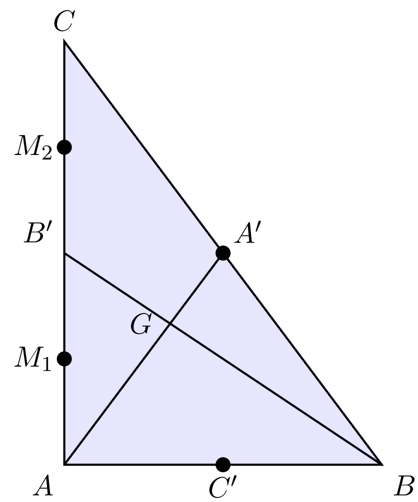
Proof using elementary geometry. The idea of the proof is to show that the point of intersection of two of the balance lines divides each of the balance lines into line segments whose lengths relate to each other 2 : 1. More precisely, on this picture of our favorite triangle



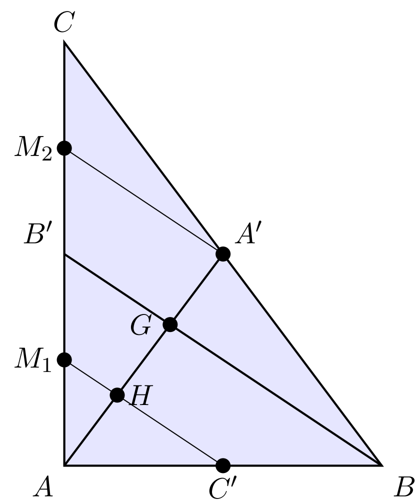
the length of AG is twice the length of GA', and, similarly, the length of BG is twice the length of GB'.

The reason why this observation will finish the proof is because the point of intersection of CC' and AA' would also have to divide AA' to line segments with length ratios $2 : 1$, hence it has to be G.

To prove our claim about the position of G on AA', consider the midpoints of AB' and $B'C$,



and draw the lines M_1C' and M_2A'.



Note that the lne M_1C' is parallel to BB' since it connects to midpoints in the triangle ABB'. Similarly, M_2A' is also parallel to BB'. Since the lengths of AM_1, M_1B' and $B'M_2$ are all equal, and the lines M_1C', $B'B$ and M_2A' are parallel, these line will cut out segments of equal lengths from AA'. This means exactly that AG is twice as long as GA'. □

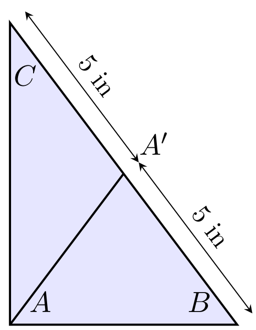
So neither of these proofs correspond to the interest and discovery of balance lines and center of gravity.

While I don't think kids need a proof, if they are interested in finding out why connecting a corner with the midpoint of the opposite side, here's what one could tell them.

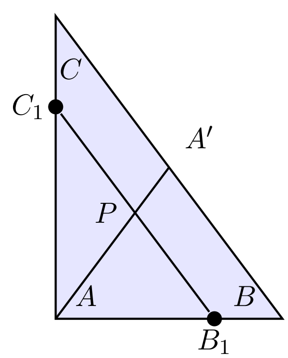
Before the proof, the kids should be reminded that if they want to balance a pencil on the tip of their finger, they have to place their finger in the middle. For another example, a seesaw is always underpinned in the middle.

Kids also need to know about *similar* triangles.

How can we understand that AA' is a balance line in our favorite triangle



The main idea is to be able to see that the triangle can be viewed as a collection of pencils or seesaws. To begin, note that if BC was a pencil, we could balance it at A'. Now take *any* line which is parallel to BC, and suppose they intersect the triangle at the points B_1 and C_1



Since the triangles ABC and AB_1C_1 are similar, the lengths of B_1P and PC_1 are equal to each other. As a consequence, we can balance the B_1C_1 "pencil" at P. Clearly, drawing all lines parallel to BC, the resulting pencils make up the whole triangle, and are balanced at their point of intersections with AA'.

3 Concept: functions, variables

3.1 Purpose

The *general* purpose of this section is to demonstrate that most of the concepts and formalisms of mathematics have been invented to describe *concisely* what we see and experience in our surroundings. We can view most of the notations in mathematics not as some abstract building blocks, but simply as abbreviations.

A very common mistake in teaching mathematics is to teach too many concepts, too many definitions. As in regular language, abbreviations such as FBI or NPR are useful, but using too many of them inhibits fluent communication.

As a result, a teacher should teach only the most fundamental concepts, definitions. It's not necessary to define something precisely in order to understand it. The often emphasized "vocabulary assignments" should be avoided at all cost.

The *specific* purpose here, in this section, is to introduce the concept of a function and, at the same time, of a variable so that the following will be easy for the kids to grasp, without struggling with the meaning behind the formalism.

formula for a function: they'd understand that when they see, say, $f(x) = x^2$ or $x \overset{f}{\mapsto} x^2$, they'd know, it's a concise way of writing "let's assign the square to every number, and call this assignment f."

evaluation of a function, "plug in": If the kids are asked

> *"Find $f(-2)$ when $f(x) = x^2$."*

they'd know exactly what to do (replace x by -2) and what they are doing (they work out what the abbreviation $f(x) = x^2$ stands for when $x = -2$).

graph of a function:

"vertical line test":

3.2 Incorrect ways

Introducing functions via their graphs, their formulas or even as assignments of numbers to numbers is not effective. Even in college, many kids have problems articulating what their understanding of a function is: they try to recall a definition they memorized and they often have no idea what they are doing when they are "evaluating" a function at a point or when they are "plugging in".

I think the two most common mistakes that are made in introducing functions are the same as introducing *any* mathematical concept.

1. The teacher tries to give the definition early on, implicitly assuming that proper understanding of mathematics requires definitions. Not true at all. In fact, one of the principle ways of making math look dry, overly pedantic and complicated is through endless "math vocabulary" assignments. Do kids need the definition of an apple in order to be able to identify it, taste it, enjoy eating it? Do kids (or anybody else) need to know grammar to be able to speak and understand a language?

2. The early definition is followed by a good number of examples on the use of the concept. This doesn't promote understanding, this is simply the most basic way of brainwashing: if we repeat something often, the feeling of *familiarity* we develop towards it we mistake for understanding.

Let us see some concrete examples for incorrect introduction of functions. To understand best why they are incorrect, imagine an average 5th or 6th grader, and imagine they are reading these for the first time.

- At this webpage, we can read the following classical definition,

  > A **function** is a rule that relates how one quantity depends on other quantities.

  It then proceeds showing examples of defining functions via formulas. Kids learning about functions this way will end up thinking of functions as equations where on the left of the equality sign"=" we see $f(x)$ or y and on the right side there is a formula. So this definition combines rules (formulas), relationships and dependence to make up the concept of a function. Since this is foggy and complicated, the hope is that lots of examples will enforce correct use of the concept.

  This reminds me of the concept of the Trinity: it's foggy, complicated, but priests and parents hope that if kids are made to refer to it enough many times, they get used to it, and even believe, they have an understanding of it.

- This blog introduces functions as machines, as if a function was doing something with the input and then spits out the output. Though this dynamic definition of a function is a powerful and useful idea, it's more complicated then it needs to be in an introduction to functions: It uses words like "input", "output", "rule" "machine" that are not necessary for the function concept, and each may require some explanation of their own. The blog's treatment also assumes that kids already are fluent with using variables such as "x". More importantly, the "function-machine" is just a reformulation of the idea that a function is a rule, given by formulas.

  I think the most serious mistake this teacher makes is that she doesn't let the kids go home with this function-machine idea, but she crams in other ways of "giving" functions, like by tables or their graphs.

- This webpage gives this definition of a function,

  > A function f of a variable x is a rule that assigns to each number x in the function's domain a single number $f(x)$. The word "single" in this definition is very important

  The outlined treatment is a combination of the "by formula" and the "machine" definition of a function. While the author suggests quite a lot of preparation before giving the definition of a function, he thinks, it's important to formalize terminologies like *independent variable* which are not only outdated but unnecessary in a first treatment.

3.3 Taking time, forget rules, postpone formulas

Some concepts in mathematics are easiest to understand if we follow its historical development instead of jumping into its modern mathematical treatment. Many concepts and results in geometry are easiest to understand in their original physical contexts. For example, finding the center of gravity of a triangle is such, as we saw it in section 2. Another example is calculus. While the intuitive understanding of instantaneous velocity, as a limit value of average velocities, poses no serious problems to a college student, the "$\epsilon - \delta$" definition of limit is very difficult to digest, and is necessary to understand it only for a mathematician.

Sometimes it also happens that the abstract development of a concept is much clearer and easier to understand than it was at the time of Euler or Newton. The *function* concept is such an example, thanks to the work of Cantor.

But we don't want to start with the definition. We want to start with examples.

3.3.1 Eye colors

For simplicity, let's assume, we only have the following four kids in the class: Al, Brooke, Curt, Dora. Pair them up, and tell them

"Tell your partner the color of your eyes, and ask her or him to confirm it."

Put the following two headers on the board,

Name	**Eye Color**

and then ask the kids, one by one[2], to put their names and eye color on the board. Something like this,

Name	**Eye Color**
Al	blue
Brooke	brown
Curt	green
Dora	brown

Chances are that the writing is not too neat on the board, so make it clear which eye color belongs to which kid with arrows

[2] With 30 kids in the class, this may appear tedious, but kids get occupied with checking each others's eye color. If it happens that kids all have the same eye color, choose something different, like color of their shirts. Feeling a bit tedious about this is actually serves a purpose, as we will see.

Name		**Eye Color**
Al	$\longmapsto$	blue
Brooke	$\longmapsto$	brown
Curt	$\longmapsto$	green
Dora	$\longmapsto$	brown

3.3.2 Moms' names

Now ask the kids to come up with something similar, like their hair color or how many siblings they have (possibly 0) or their mothers' names. Suppose they chose their moms' names. Then ask the kids not only write their moms' names but also draw the arrow from their names to their moms'.

Name		**Mom's name**
Al	$\longmapsto$	Julie
Brooke	$\longmapsto$	Mary
Curt	$\longmapsto$	Ellie
Dora	$\longmapsto$	Katie

3.3.3 General descriptions, assignments

Ask now the kids

"Can you describe in one sentence what we did when we said Al's eyes are blue, Brooke's are brown, etc?"

Kids eventually will says something like

"We wrote down what the eye color of each us was."

Well, in more mathematical language, we say, to each kid, we *assigned* her or his eye color. So we assigned *blue* to *Al*, *brown* to *Brooke*, etc. In general, we say, *"To each student, we assigned her or his eyecolor."*

We can indicate this assignment as

$$\textit{Student} \longmapsto \textit{Eye Color}$$

This generic description of the assignment is much more concise than the concrete one

$$\begin{aligned} &\text{Al} &&\longmapsto \text{blue} \\ &\text{Brooke} &&\longmapsto \text{brown} \\ &\text{Curt} &&\longmapsto \text{green} \\ &\text{Dora} &&\longmapsto \text{brown} \end{aligned}$$

We can get the concrete description from the generic one by *substitution*: replace *Student* by Al and *Eye Color* by blue, to get,

$$\text{Al} \longmapsto \text{blue}$$

replace *Student* by Brooke and *Eye Color* by brown, to get,

$$\text{Brooke} \longmapsto \text{brown}$$

etc.

Let's now go through the same path to get to the "student-mom's name" assignment. Ask

"Can you describe in one sentence what we did when we said Al's mom's name is Julie, Brooke's mom's name is Mary, etc.?"

Kids' answer will come easy

"We wrote down what the mom's name of each of us was."

Then get them to come up with the generic assignment

$$\textit{Student} \longmapsto \textit{Mom's name}$$

Finally, ask them to use substitution to get some of the concrete assignments like

$$\text{Al} \longmapsto \text{Julie}$$

or

$$\text{Brooke} \longmapsto \text{Mary}$$

3.3.4 Homework

Ask the kids to come up with 3 assigments involving family members, and describe them both in the generic and concrete ways. Possibilities: favorite food, TV show, movie, book, song, singer, mass media (facebook, snapchat).

3.3.5 Class 2: number-assignments

After asking the kids to talk about what kind of assignments they came up with which involve family members, tell them, in math, we mostly make up assignments involving numbers. For example

$$\begin{array}{ccc} 1 & \longmapsto & 2 \\ 2 & \longmapsto & 4 \\ 3 & \longmapsto & 6 \\ 4 & \longmapsto & 8 \\ \vdots & & \vdots \end{array}$$

It's worth asking the kids,

"How long will it take to write down this whole assigment, for every number? When can I stop writing it up?"

The answer, of course, is *never*.

Anyhow, the the kids would soon say, the generic description of this assignment would be

$$\textit{number} \longmapsto \textit{twice the number}$$

How can we make this even shorter? We could abbreviate "*number*" by its initial n and instead of "*twice*", use the mathematical notation for multiplication

$$n \longmapsto 2 \cdot n$$

The great thing about this generic description is that it refers to *infinitely many* concrete assignments. So a generic description is much more powerful than a usual abbreviation: it's an abbreviation for infinitely many things which we could never ever finish writing up! For example, when $n = 1000$, it refers to the assignment

$$1000 \longmapsto 2 \cdot 1000 = 2000$$

Not only that, the assignment makes sense even in case when n refers to a non-integer number, like $n = 1.2$,

$$1.2 \longmapsto 2 \cdot 1.2 = 2.4$$

We could call this assignment the "*multiplication by two*" assignment. But the kids should suggest a single letter for its name, say, they suggest the letter t from the beginning of the word "*t*wo". We can incorporate the name t into the generic assignment as

$$\mathrm{n} \overset{t}{\longmapsto} 2 \cdot n$$

The kids now should be asked to come up with the generic description for

$$\begin{array}{ccc} 1 & \longmapsto & 3 \\ 2 & \longmapsto & 6 \\ 3 & \longmapsto & 9 \\ 4 & \longmapsto & 12 \\ \vdots & & \vdots \end{array}$$

They'll get something like

$$\mathrm{n} \overset{h}{\longmapsto} 3 \cdot n$$

where the h is coming from "t*h*ree".

The next one may require more time to figure out.

$$
\begin{aligned}
1 &\longmapsto 1\\
2 &\longmapsto 4\\
3 &\longmapsto 9\\
4 &\longmapsto 16\\
\vdots & \qquad \vdots
\end{aligned}
$$

It is

$$\text{n} \xmapsto{s} n^2$$

where s is from "$s$quare".

3.3.6 Homework for class 2

Problem 3.1. Find the general assignment and name it with a single letter.

$$
\begin{aligned}
1 &\longmapsto 1\\
2 &\longmapsto \tfrac{1}{2}\\
3 &\longmapsto \tfrac{1}{3}\\
4 &\longmapsto \tfrac{1}{4}\\
\vdots & \qquad \vdots
\end{aligned}
$$

Problem 3.2. You are given the assignment

$$\text{n} \xmapsto{q} 2 \cdot n^2$$

What does q assign to 3, -3, $\frac{1}{2}$, 0.4?

3.3.7 Class 3: functions, $f(x)$ notation

After warming up with discussing the homework problems, we tell the kids that this class is about a series of admissions. The first admission is that numbers are most often denoted by x, and assignments are called f, so instead of

$$\mathrm{n} \xmapsto{t} 2 \cdot n$$

we often see

$$x \xmapsto{f} 2 \cdot x$$

Don't ask about the use of x, but there is an explanation for the preference of f: it stands for the word "*function*". Which leads us to the second admission: assignments are called functions in mathematics.

The last admission will make the most sense: while our arrow-notation for an assignment is descriptive, mathematicians like to use an even sorter notation: they often write $f(x) = 2 \cdot x$ instead of

$$x \xmapsto{f} 2 \cdot x$$

or $f(x) = x^2$ instead of

$$x \xmapsto{f} x^2$$

As a result, if $f(x) = 2 \cdot x$, we then write

$f(1) = 2 \cdot 1$	instead of	$1 \longmapsto 2 \cdot 1$
$f(2) = 2 \cdot 2$	instead of	$2 \longmapsto 2 \cdot 2$
$f(3) = 2 \cdot 3$	instead of	$3 \longmapsto 2 \cdot 3$
$f(4) = 2 \cdot 4$	instead of	$4 \longmapsto 2 \cdot 4$

Kids should now solve the following problems in class

Problem 3.3. Suppose we have the assignment $x \overset{f}{\mapsto} 3 \cdot x$. Write it in "$f(x) =$" form, and then find out what numbers are assigned to 1, -1, $\frac{1}{3}$, $-\frac{4}{3}$. In other words, find out what $f(1)$, $f(-1)$, $f\left(\frac{1}{3}\right)$, $f\left(-\frac{4}{3}\right)$ are.

Problem 3.4. Suppose we have the assignment $x \overset{g}{\mapsto} x^2$. Write it in "$g(x) =$" form, and then find out what numbers are assigned to 1, -1, $\frac{1}{3}$, $-\frac{4}{3}$. In other words, find out what $g(1)$, $g(-1)$, $g\left(\frac{1}{3}\right)$, $g\left(-\frac{4}{3}\right)$ are.

3.3.8 Homework for class 3

Problem 3.5. Suppose $f(x) = \frac{1}{x}$. Write this function with the arrow notation. Then complete the chart

$$10 \longmapsto$$

$$-4 \longmapsto$$

$$\tfrac{1}{2} \longmapsto$$

$$-\tfrac{2}{3} \longmapsto$$

Problem 3.6 (Challenging)**.** Consider the "eye color" assignment

$$\begin{array}{lcl} \text{Al} & \longmapsto & \text{blue} \\ \text{Brooke} & \longmapsto & \text{brown} \\ \text{Curt} & \longmapsto & \text{green} \\ \text{Dora} & \longmapsto & \text{brown} \end{array}$$

Write down the generic assignment for it, name the assignment with a single letter, then write it in "$f() =$" form. Finally rewrite each of the concrete assignments in $f() =$ form. (Solution: call the function E, and then the generic form may look like $E(name) = color$. The specific forms look like $E(Al) = blue$, $E(Brooke) = brown$, etc.)

3.3.9 Class 4: visualization of functions, graphs

Not sure, if one should do this, or recommend to do something else, instead, and leave functions for a while.

3.3.10 Excuse

We, of course could have started with introducing and practicing notations such as $f(x) = 2x$, $g(x) = x^2$. This yields fast treatment of material needed for tests. But would kids see that this notation is simply an abbreviation, and not some abstract stuff unique to mathematics?

OK, but why not at least start immediately with number functions? Because the kids then won't realize that the function concept is used every day such as when we survey a group of people for their favorite food or TV show.

4 Algorithm: adding fractions

To come after Holidays.

5 The Puppet Master

I think of the math teacher as a puppet master and math is the puppet. The puppet should take center stage, that's what the kids should really relate to with affection, and appreciation, but of course the puppet master is the human connection to the world of puppets.

There should be no reason for a teacher to steal the show from math. And I am not talking about just bad, boring teachers, but highly rated ones as well: There was this math prof at Northwestern who won the best teacher award during the two years I was there. I asked the kids what they liked most about his teaching. They said: "When he brought in a watermellon, and cut it in half with a single strike of a butcher's knife." But the students didn't remember what the watermelon "experiment" was supposed to demonstrate.

To me, the perfect puppet master math teacher is the young guy in the youtube video I already showed you. He is cool, entertaining, but he still manages to stay in the background.